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TIGRO-M: A Program for Automatization of Multiplication Facts

—An Intervention for Children with Math Difficulties

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Abstract

This study evaluates the effectiveness of a new intervention program with primary school children who show difficulties in solving simple multiplication problems. 64 fourth-grade children completed an eight-week long training with a pre-, post-, and follow-up test. Performance on a standardized test for arithmetic operations and a computer test were used to measure children's knowledge. Results show the overall and long-term effectiveness of a holistic approach to multiplication fact training. All participants of the experimental groups show a significant increase in correct answers and faster reaction times, with the effects being stable over time. The program helped children build a structured multiplication fact network, with untrained facts being solved more quickly and correctly after the intervention.

Keywords

Multiplication Problems, Intervention, Arithmetic Fact Training, Math Difficulties, Learning Disabilities, Elementary School Children

1. Introduction

Solving multiplication problems is a basic skill, children learn in their early mathematics education. Learning all the strategies and rules is usually associated with great effort and the road to an automatic retrieval is long, yet a desirable accomplishment for elementary school children (Burns, Ysseldyke, Nelson, & Kanive, 2015). Proficient individuals are able to effortlessly access the knowledge from long-term memory and have the ability to use procedural and conceptual knowledge. Only in the course of time children become able to use their long-term memory to get the declarative representations of numeral number facts (Ashcraft, 1987) and they shift to a more strategic approach (Siegler, 2007). These skills develop through repeated practicing of already learned facts (Allen-Lyall, 2018; Delazer et al., 2004). Using effective associative networks in elementary school is seen as a main predictor of prospective performance in mathematics (Kaye, 1986). An associative network is an effective connection between individual multiplication problems and its stability is influenced by a good procedural and conceptual knowledge. The retrieval of facts from long-term memory is determined by the strength and stability of the representations or networks (Kaufmann, Handel, Delazer, & Pixner, 2013; Roussel, Fayol, & Barrouillet, 2002). The concepts of procedural and conceptual knowledge are further explained in Chapter 1.1. Solving multiplication problems. Fluency in basic arithmetic skills (especially multiplication) interferes with being able to understand and solve even more complex problems in elementary school and secondary education (Codding, Burns, & Lukito, 2011). Internalized and automated facts further allow associations between mathematical concepts, such as multiplication and division (Chapin & Johnson, 2006). Hence, for the understanding of division, for the learning of fractions and also for further tasks of combining, multiplications provide an important foundation (Merdian, Merdian, & Schardt, 2012). Accordingly, a poor understanding of multiplication and a lack of or deficient multiplication fact knowledge can prevent students from reaching higher levels of mathematics (Lee, 2007). In this case, the focus should not only be on the cognitive deficits that build-up, but also take into account that repeated experiences of failure result in a negative attitude toward arithmetic in many affected children, which can not infrequently lead to school anxiety and general school refusal (Krinzinger et al., 2009).

Devine, Soltész, Nobes, Goswami and Szücs (2013) reviewed prevalence studies and concluded that about 5% - 6% of the population suffer from developemental dyscalculia. In 2018 Morsanyi, van Beers, McCormack and McGourty (2018) found out that 6% of 2421 primary school children had persistent, severe difficulties with mathematics. Haberstroh and Schulte-Körne (2019) are expecting a range from 3% - 7% in the overall population. Even more children need targeted support (Huijsmans, Kleemans, & Kroesbergen, 2021). Also children that are not formally diagnosed with developmental dyscalculia but would be considered as children with poor numeracy skills/math difficulties are considered to show lower performances in later arithmetic. Many children with mathematical learning problems have not achieved automaticity in basic arithmetic facts and have problems retrieving them. This is particularly problematic in that the process of multiplication can be error-prone even in proficient adults. Thus, it is important to create a secure basis in the multiplication process early on, especially for children with math difficulties.

1.1. Solving Multiplication Problems

Multiplication facts are assumed to be stored in associative networks (Ashcraft, 1987). Arithmetic facts can be stored as individual representations or as general rules (such as multiplying with 0 or 1; Sokol, McCloskey, Cohen & Aliminosa, 1991). Storing and retrieving multiplications becomes easier and more effective with connections between individual multiplication problems. The stability of the networks is highly influenced by a good procedural and conceptual knowledge. Conceptual knowledge includes the knowledge that 3 × 4 can also be calculated as 4 + 4 + 4, while procedural knowledge enables to build associations and rules, for example for the operation N × 0 is always 0 and the result or 3 × 4 leads to the same result as 4 × 3. Releasing the facts from long-term memory is dependent of the strength and stability of the representations (Kaufmann et al., 2013). The more frequently an arithmetic operation is trained, the better it is stored in our memory and the more easily and faster it can be retrieved (see also Allen-Lyall, 2018). The associative networks for multiplication facts not only hold one form of multiplication problem but also complementary operations, meaning tasks with changing operand (i.e. 2 × 3 or 3 × 2) (Butterworth, Marchesini, & Girelli, 2003; Campbell & Robert, 2008). In a study with adult students Campbell and Robert (2008) found clear transfer effects from a trained task (2 × 3) to a specific complementary operation (3 × 2), shown as improved reaction times.

The numerous needs that are necessary when solving multiplications can be error-prone and are sensitive to interferences (De Visscher & Noel, 2014). Therefore, it is important that associative networks for multiplication problems are built stable and structured. The idea of associative networks is supported by Domahs, Delazer and Nuerk (2006), who reported that the majority of erroneous responses reflect operand errors, belonging to the multiplication table of one of the operands (e.g. 3 × 7 = 28; 3 × 7 = 18).

For solving different multiplication problems children use different strategies (Siegler, 1988). When gaining skills and knowledge about how to solve mathematical problems children develop and internalize a wide spectrum of arithmetic procedures and strategies. They learn how to select as well as detect and they also use the conceptual knowledge for generating new methods of problem solving and discover solutions (Mabott & Bisanz, 2003). Amongst others there is often used the repeated addition or the direct retrieval from long term memory. LeFevre, Bisanz, Daley, Buffone, Greenham and Sadesky (1996) attended the question how to solve simple multiplication problems. There are different methods of how children generate the correct result. Possible options are the direct retrieval, the use of rules, as well as repeated addition, the derivation of already existing facts (e.g. by doubling them) (McCrink & Spelke, 2010) or number series. Children with dyscalculia often have problems building up and/or retrieving their arithmetic knowledge so that they often refer to back-up strategies (Baroody, 2003). Accordingly, dyscalculia interventions tend to promote alternative strategies for solving arithmetic problems.

1.2. Effectiveness of a Multiplication Training

Learning multiplication facts is the focus in most educational settings and previous intervention programs. Dedicated trainings seem to achieve favorable results in the storage and recall of arithmetic facts (Allen-Lyall, 2018; Nelson, Burns, Kanive, & Ysseldyke, 2013; Wong & Evans, 2007). While current intervention studies with facts focus on the mere memorizing of the facts (Burn et al., 2019; Sleeman et al., 2021), conceptual understanding is often left unaddressed.

Children with mathematical difficulties and dyscalculia do not easily build up associative networks and storage that eases the retrieval of arithmetic facts (Kroesbergen & van Luit, 2003). They also perform poorly in all areas of mathematics and have problems with basic arithmetic operations (Haberstroh & Schulte-Körne, 2019). Therefore, they need special assistance, since the automatization of basic arithmetic facts is the basis for understanding more difficult arithmetic problems (McCallum, Skinner, Turner, & Saecker, 2006). Since there are always subgroups with specific problem areas, it is particularly important that all parts of multiplication learning-procedural knowledge, conceptual knowledge and fact retrieval—are implemented in a intervention program (Gomides, Martins, Alves, Júlio-Costa, Jaeger, & Haase, 2018). Based on these assumptions and as a continuation of the previous intervention programs, we additionally focus on procedural knowledge and especially conceptual knowledge besides pure fact retrieval in the current study.

A development in strategy use was the basis for previous intervention studies for children with math difficulties (Wood, Frank, & Wacker, 1998; Zhang, Xin, Harris, & Ding, 2014). Learning multiplication tables in categories (zeros, ones, doubles, fives, nines and the remaining) let children with difficulties focus more on the strategy they had to use (Wood et al., 1998). Additionally, the use of visual illustrations seems to be efficient in children with learning problems (Huang & Chao, 1999). Zhang and colleagues (2014) suggested that a strategic training program that involves students using different strategies and asks for self-explanation improves the problem solving accuracy in children with math difficulties. This goes along with the need for children to build stable multiplication networks, they can access while solving arithmetic problems.

Children with dyscalculia and math difficulties are assumed to not have a stable procedural and conceptual knowledge of the multiplication operation and therefore lack good structured associative network with multiplication facts (Kroesbergen & van Luit, 2003; Siegler, 1988). Therefore it is likely that they lose new learned facts more easily and are unable to transfer from one task to another due to missing associative connections. The availability of learned facts—which is determined by the associative strength between problem and result—plays a critical role (Shrager & Siegler, 1998). If simple multiplications have already been practiced, the associative strength is high so the task can be solved faster. As already mentioned Delazer and colleagues (2004) showed that intensive practice of already learned facts led to an improvement in speed and accuracy of fact retrieval. As such, automatization is an effective way of validation for mathematical knowledge (Symonds & Chase, 1992). Well stored and automated facts are also believed to transfer to related operations. Rickard and Bourne (1996) found out that exercises of multiplication with changed operands (the exercises in the test were complementary to the exercises which were trained) can be solved as quickly as the trained exercises (see also Butterworth et al., 2003). They assume, that there are two different phases to the multiplication process, the so-called phase of perception and the phase of cognition. The phase of cognition contains an abstract representation which isn't depending on the chronology of the operands (van Galena & Reitsma, 2011). Therefore, complementary operations should be comparable to specifically trained facts, as long as they are stored in associative networks.

Considering the training of arithmetic facts, the speed of recall can clearly be increased by simply memorizing facts. Yet, for the integration and generalization of fact knowledge, comprehension of the underlying calculation procedures and conceptual knowledge are essential (see Baroody, 2003). To effortlessly retrieve multiplication facts, an adequate automatization of these facts is needed (Burns, 2005). Therefore, the cornerstone for any effective multiplication training is the balance between necessary comprehension of procedural and conceptual aspects and adequate automating of multiplication facts—all being implemented in our training.

Generally, it is important to note for an intervention, that a light impairment of mathematical skills may be caused by deficits of the memory concerning acquisition and remembering (Kroesbergen & van Luit, 2003). To guarantee the effectiveness of a training, an adequate timing as well as an adequate setting has to be considered. Furthermore an intervention in one-to-one sessions was observed to be most effective (Kroesbergen & van Luit, 2003). Another guarantor for the effectiveness is the kind of instruction been provided. Kroesbergen and van Luit (2003) showed that a direct instruction (i.e. to explicitly clarify the conceptual way) is the most appropriate kind for learning basic arithmetic skills.

The focus of this study will thus be on the evaluation of a training program (TIGRO-M) based on a holistic learning approach of multiplication facts in children with difficulties in mathematics.

1.3. TIGRO-M

In our newly developed training (TIGRO-M: Pixner & Dresen, 2021), we will focus on the evaluation of its effectiveness in improving multiplication performance in children with difficulties solving those tasks. The intervention program is not only based on the improvement of fluent fact retrieval but will holistically include components of fact acquisition, such as procedural and conceptual knowledge. TIGRO-M is therefore based on three columns: procedural understanding, conceptual understanding and fact memorization. The combination of all three components should ensure that the facts are memorized in the long term, i.e. that they are not simply learned by rote and then lost again. In addition, this approach should also ensure that learned knowledge is applied in other domains and leads to improvement also in non-trained domains. This could be proven by the expected transfer effects. We combined the third column (fact memorization) with color-number association and ultimately tested one training group with and one without the addition of color. Besides the specific intervention effect in both experimental groups, the consistency of the improvement/stability of learning are also in focus. In the case of a good conceptual and procedural understanding, we expect the built facts to be reflected in strong associative networks that will also provide information for unlearned facts.

1.4. Color-Number Association in an Intervention Setting

The learning process can be optimized with concrete illustrative material or a key stimulus. Mnemonic strategies are usually words, sentences or rhymes that improve the storage and recall of facts (Nelson et al., 2013; Test & Ellis, 2005) but can also be colors. Based on the phenomenon of synesthesia, where numbers are naturally associated with colors (Kadosh, Henik, & Walsh, 2009, Spector & Maurer, 2013), for example 2 is always colored green, Domahs, Lochy, Eibl and Delazer (2004) created a simulated scenario. In a single-case study with a brain-damaged patient, the storage and recall of facts was improved with the use of colors. The key stimulus was presented in different colors (comparable to reported color-associations of synesthetes) to facilitate building up and retrieving multiplication facts. A well-trained association between a color and a specific number seemed an optimal key stimulus for the built up and retrieval of simple multiplication facts (Domahs et al., 2004). Based on these findings the current intervention study makes use of a color-number association and allows a closer look at whether it helps with automatization and retrieval of multiplication facts in children with math difficulties.

2. The Current Study

The present intervention study examined how far the retrieval and the storage of arithmetical facts of simple multiplications can be improved for children with difficulties while using conventional studying techniques with specific methods of support. We aimed to create a training program that is built on a holistic approach to multiplication fact storing and retrieval. The intervention program (TIGRO-M: Pixner & Dresen, 2021) rests on three columns for internalizing and automating multiplication facts. The first column comprises conceptual comprehension. Children should learn to understand that the arithmetic operation of multiplication can be seen as repeated addition (for example 3×4 is the same as $4 + 4 + 4$). The second column focuses on procedural understanding by conveying rules and regularities found in multiplication (e.g., multiplying by 0, 1, and 10; complementary problems, e.g., $2 \times 3 = 3 \times 2$ etc.). All procedural and conceptual training steps were practiced using visual illustrations (see Huang & Chao, 1999). Thereby, it is intended to contribute to building stable associative networks for the multiplication facts. The third column is dedicated to the automatization of multiplication facts to increase automaticity through associative strength within the network. Associations between a specific problem and its correct result should be stored in long-term memory by playful repeated learning and specific color-coding of results.

Against the theoretical background we firstly expect the intervention with our training program (TIGRO-M) to be effective in improving multiplication, as we build it on all three important aspects of solving multiplication problems (procedural and conceptual knowledge, fact memorization). With respect to the effectiveness of our training program, we also aim to show that effects are specific to improving multiplication problems (and not to subtraction problems, that were not specifically trained). We also want to see whether the number-color association has an additional effect on training success. Irrespective of the training modality ("regular" black and white training or with number-color connection) we expect stability of improvements. Additionally, we would like to take a closer look at whether a connection of the learned facts to unlearned facts occurs due to our training approach. If there is a strong network association of the multiplication facts, then this should be reflected in transfer performance on untrained items (e.g. shown by Campbell & Robert, 2008). This means, once the children have acquired a good procedural and conceptual understanding as a result of the intervention and have built up a stable, well-organized network of facts, they should be able to recall the non-trained complementary items just as well as the trained items. In terms of evaluation of M-TIGRO the aim of this study is to find out if this intervention has positive effects on multiplication facts training of children.

3. Method

3.1. Participants

The sample comprises 64 fourth-grade elementary school children with difficulties in solving multiplication problems. Participants were selected based on their performance in the subtest "multiplication" of the Heidelberger Rechentest (HRT 1-4; Haffner, Baro, Parzer, & Resch, 2005). Only children who scored below percentile rank (PR) 30 were chosen for the intervention (adapted from Geary, Hamson & Hoard (2000), who allowed children to PR 35 in their intervention study). We intentionally included not only children with diagnosed dyscalculia, but also those with a weakness in multiplication, as these children also need focused support. Often children can compensate their deficits for a long time with other competencies and are not immediately noticed as weak in arithmetic (Huijsmans et al., 2021). Nevertheless, these children also highly benefit from specific support. 68.8% of participants were native German speakers. The remaining participants were bilingual, with German and one other language. Children were randomly assigned to one of three groups, either one of the two intervention groups or the control group. See **Table 1** for details of descriptive statistics of the groups including information on n, age, and sex.

Table 1. n, distribution between the sexes and age within the groups.

Group	n	Sex	Age in month (Sd)
Control group (CG)	20	Male: 45.0%	118.20 (6.01)
Intervention group color (E1)	23	Male: 47.8%	120.39 (9.22)
Intervention group black/white (E2)	21	Male: 42.9%	119.52 (7.59)

Parents were informed and had to provide written consent for their children to participate in the study. This study was approved by the Research Committee for Scientific and Ethical Questions of the UMIT Tirol—The Tyrolean Private University and the "Bildungsdirektion" Tyrol, Austria.

3.2. Procedure

Intervention- and test-dates were one-on-one session. Each child performed a pre-, post-, and follow-up test, as well as an 8-week intervention program between pre- and post-test. Children were trained according to a detailed manual which contains explanations on each and every multiplication table. Children were trained one session per week lasting about 45 to 50 minutes. The control-group received a standardized orthographic training (Marburger Rechtschreibtraining; Schulte-Körne & Mathwig, 2009) to the same extent. For the duration of our intervention, parents and teachers were explicitly instructed to avoid any special training on multiplications. Teachers specifically avoided repeating multiplication tables in the classrooms. Also, the parents were instructed to not practice multiplication problems at home. The follow-up test took place 8 - 10 weeks after the post-test. To evaluate the stability of the effects of learning, experimental groups did not receive further training during this period. For ethical reasons the control-group completed the same intervention program (TIGRO-M), the other participants got initially, in this time period.

3.2.1. Intervention with TIGRO-M

Our intervention program is called TIGRO-M (training of arithmetic skills—multiplication, in German: **T**raining **i**n **G**rund**r**echen**o**perationen—**M**ultiplikation; Pixner & Dresen, 2021). The training involved a weekly session over the course of eight weeks. Except for the first and the last session, training units followed the same procedure (see Table 2). The first unit was used to establish a trustful relationship with the children and allowed to focus on all multiplication rules more explicitly (multiplication with 0, 1, and 10). In the second session facts of the multiplication of 2 and 5 were trained. From the third to eighth session multiplication tables of 3, 4, 6, 7, 8 and 9 were focused on. The last session involved a repetition of all multiplication tables with different games, for example memory games, where pairs of multiplication problems and correct solutions have to be found.

The exact procedure was described in detail in a manual, so every instructor doing the intervention was able to follow the same modus operandi. All instructors additionally received a briefing before the training to ensure objectivity.

Table 2. Structure of the training sessions.

	Structure of training session
1. part	Repetition of facts from the last unit
2. part	Procedural and conceptual knowledge games
3. part	Learning of 5 new facts
4. part	Games for automatization of new facts
5. part	Repetition of new and old facts combined

Note: All parts took approximately 10 minutes each.

It is important to note that only half of the facts were trained for every multiplication table (cf. Butterworth et al., 2003). All complementary problems for our chose items were left out. Multiplication tables had the same level of difficulty, as we balanced problem size (e.g., 3 × 4 was trained, whereas 4 × 3 was not) between all items. The mean problem size result was 32.67 for trained facts and 32.42 for untrained items.

The three main columns the intervention program was built on were present in every intervention unit and were used on every multiplication table. Regarding the first column, the procedure of multiplication was introduced. We implemented this with repeated addition. For example, pictures of dices (e.g., 7 dices with 5 dots on it) were used, for which children had to decide which multiplication problem they represented. This was internalized by playful learning in the following. The second column focused on conceptual knowledge. Rules and regularities multiplication problems (e.g., when 7 × 5 = 35, then the result of 8 × 5 is larger by exactly 5) were conveyed. To illustrate this, dice pictures were arranged linearly to demonstrate that 8 × 5 is 7 × 5 + 5. In addition to pictures of dices also everyday objects were used for visualization, for instance twin cherries for the 2 multiplication table, a cone with three balls of ice cream for the 3 multiplication table, tires of a car for the 4 multiplication table, fingers of a hand for the 5 multiplication table, an egg box for the 6 multiplication table, days of the week for the 7 multiplication table, legs of a spider for the 8 multiplication table, and pins of skittles for the 9 multiplication table. The third column involved automatization of arithmetic facts based on the first two columns. Therefore, problems were repeated several times and thus automated while playing board games, memory games and sorting exercises. For our purpose we used common games like simple memory and materials like dices and adapted them. The game "Memory" consisted of 10 cards. 5 of them with multiplication problems and 5 with the matching results. The cards were mixed and the players took turns to turn over 2 cards each until all pairs were found. In another game, all problems from one multiplication table were written down one after another on a board. The children moved around the game board with a figure on the basis of a number they threw on the dice. They were then allowed to solve the problem which they landed on.

To see whether color plays a role for fact automatization, we had two intervention groups (E1 and E2), that only differed in the use of specific color-number associations during the automatization process. Children in the color-association group were introduced to a connection of colors and respective numbers. Small numbers were associated with lighter color (gray for 0, yellow for 1, orange for 2, red for 3 and light blue for 4) whereas larger numbers with darker color (dark blue for 5 and light green for 6, dark green for 7, brown for 8 and black for 9, cf. Kadosh et al., 2009). Participants from the intervention group color (E1) played exactly the same board, memory and sorting-games as children from the E2 group, only with colors instead of black and white.

3.2.2. Evaluation Tasks

In all evaluation sessions (pre-, post- and follow-up tests), the same selected subtests from standardized tests for basic arithmetic operations (i.e., Heidelberger Rechentest 1 - 4; Haffner et al., 2005) were used. Additionally, a specific computerized multiplication test was included.

The post-test was conducted one week after the intervention ended. Follow-up data was obtained eight weeks after the post-test.

From the HRT 1-4 (Haffner et al., 2005), subscales addition, subtraction, multiplication, and division were conducted. For each subscale, children had to solve as many of the given arithmetic problems as possible within a time limit of 2 minutes. The number of correctly solved problems served as a dependent variable for each subscale and were used for further analyses.

The computerized multiplication test was programmed using Experiment Builder (SR Research, Ottawa, Canada). In this choice-reaction-task, children had to pick the correct one of two solution probes, that were presented one below the other to the right of the arithmetic problem, by pressing a respective button ("Z" or "N"). Arabic numbers were presented in white on a black screen (font: Courier New, bold; font size: 48). Children were instructed to answer the multiplication task as quickly and correctly as possible. Eight training items were presented before the actual experiment. After this, 66 critical items (i.e., 6 per multiplication table 0 to 10) were presented in randomized order. The task lasted about 15 minutes. Occurrence of the correct solution probe was balanced, meaning that the correct solution was presented on the upper/lower part of the screen in half of the trials each. Incorrect solution probes deviated from the correct solution by 1 or 10. Tie problems were not considered (i.e., 4 × 4). Only half of the 66 critical items were trained in the intervention, to allow a later evaluation of transfer effects.

3.3. Data Analysis

All analyses were based on the number of correctly solved items of the HRT 1-4 (Haffner et al., 2005) and response times in the computer tests. For reaction times analyses only correct results were included. Data was analyzed with the statistical software IBM SPSS Statistics 25. The p-value was set at $p \leq 0.05$.

To evaluate the intervention effects, we considered three different aspects. We first conducted two 3 (group: control group vs. intervention group 1 vs. intervention group 2) × 2 (time: pre- vs. post-test) ANOVAs, one for correctly solved items on the subtest multiplication of the HRT 1-4 (Haffner et al., 2005) and one for reaction times to evaluate general efficiency of the training. Improvements in reaction time would indicate that children are using more effective strategies, so it is important to look not only at the number of correct items but also at the improvement in RTs. With, additional pairwise comparisons between the experimental groups, we tested whether the number-color association has a specific effect on multiplication fact learning. Also, we evaluated specificity of the training by running a mixed-model ANOVA with the withing-subject factors time (pre- v. post-test) and operation (subtraction vs. multiplication) and the between-subjects factor group (control group vs. intervention group 1 vs. intervention group 2) for correctly solved items on the subtests of the HRT 1-4 (Haffner et al., 2005). We specifically chose subtractions as a comparison, since they are solved with a rather different approach than multiplications (cf. Dehaene & Cohen, 1997). To evaluate the stability of the training effects from our intervention program, we again carried out two 3 (time: pre- vs. post- vs. follow-up test) × 2 (group: intervention group 1 vs. intervention group 2) ANOVAs, one for correctly solved items and one for reaction times. We intentionally refrained from including the control group in the analysis, as participants did not receive any multiplication training at first. Subjects of the control group received the same training as intervention group 2 (without color-number association) at a later time. However, we did not assess any follow-up data for this group. Lastly, we aimed to check for transfer effects from items that were trained during intervention on multiplication problems that were not trained. For that, a 2 (time: pre- vs. post-test) × 2 (items: trained vs. untrained) ANOVA was carried out for reaction times in the computer experiment.

4. Results

4.1. Effect of Intervention

To evaluate the general training effect for the number of correctly solved items, an ANOVA for the factors group (i.e., CG vs. E1 vs. E2) and time (pre- vs. post-test) was carried out. There was a significant main effect for the factor time [$F(1, 61) = 76.38$, $p < 0.001$, $\eta_p^2 = 0.56$] (see **Figure 1** for means and standard deviations), indicating that children's performance significantly improved from pre- to post-test. Also the factor group yielded a significant main effect [$F(2, 61) = 10.51$, $p < 0.001$, $\eta_p^2 = 0.26$]. Pairwise comparisons between the groups concerning the training effect (difference between pre- and post-test) reveal a significant contrast between the control group and both intervention groups (CG and E1: $p < 0.001$, CG and E2: $p = 0.001$), while the direct comparison of the intervention groups did not reveal a significant difference ($p = 1.00$) (see also **Figure 1**). No difference between the intervention groups indicates that there is no

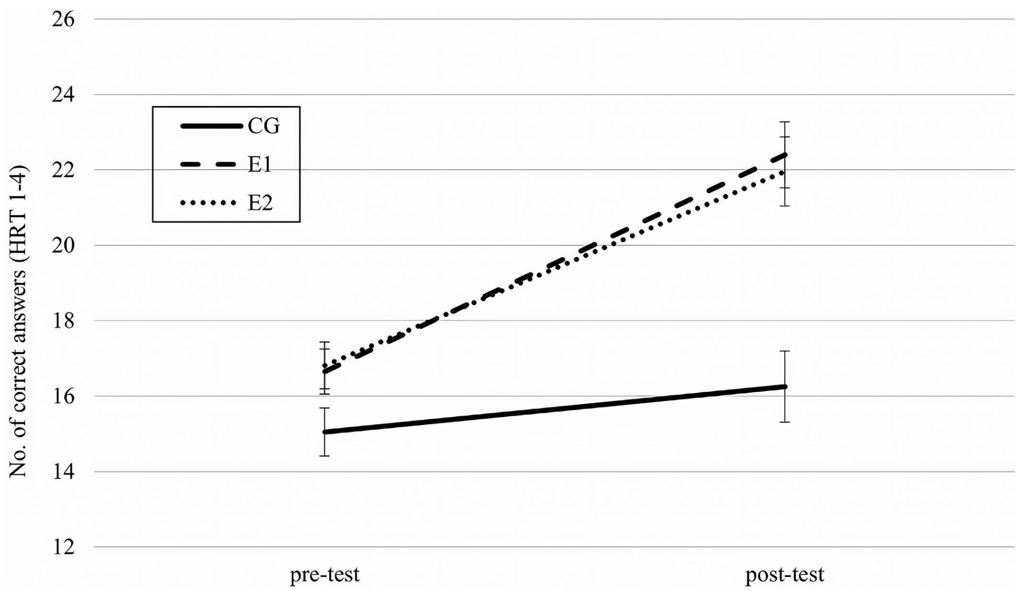

Figure 1. Performance in the subtest multiplication of the HRT separated for pre- and post-test for all groups. Error bars indicate 1 standard error of the mean (SEM).

added benefit of number-color associations in fact learning. Finally, the significant interaction between time and group [$F(2, 61) = 9.33$, $p < 0.001$, $\eta_p^2 = 0.23$] reflects the effectiveness of the training by showing that both experimental groups improved significantly from pre- to post-test, compared to the control group.

Furthermore, the overall training effect was analyzed with respect to response times on the computer test. Again, the factor time yielded a significant main effect [$F(1, 61) = 19.98$, $p < 0.001$, $\eta_p^2 = 0.25$], reflecting a clear improvement of multiplication performance in terms of speed (see **Figure 2**). Contrary to the results for correctly solved multiplication problems, we found no significant main effect of the factor group [$F(2, 61) = 0.34$, $p = 0.71$, $\eta_p^2 = 0.01$]. Yet, the interaction for time and group was significant [$F(2, 61) = 3.60$, $p = 0.033$, $\eta_p^2 = 0.11$], indicating that the decrease of response times due to the training was significantly more pronounced in the two experimental groups as compared to the control group (CG and E1: $t(41) = 2.54$, $p= 0.015$; CG and E2: $t(39) = -2.09$, $p= 0.043$). The difference between the two experimental groups was not significant (E1 and E2: $t(42) = 0.51$, $p = 0.612$). As such, the additional color-number-association did not seem to provide an additional advantage in this intervention program (TIGRO-M).

An additional analysis was conducted to point out the specificity of the TIGRO-M program to multiplication fact improvement. For that, the number of correctly solved items on the subscales multiplication and subtraction (which was not trained) was directly compared. Besides significant main effects for time [$F(1, 61) = 60.91$, $p < 0.001$, $\eta_p^2 = 0.5$] and group [$F(2, 61) = 10.02$, $p < 0.001$, $\eta_p^2 = 0.23$], the ANOVA revealed no significant main effect for the factor operation [$F(1, 61) = 51.28$, $p = 0.087$, $\eta_p^2 = 0.05$]. In line with previous findings,

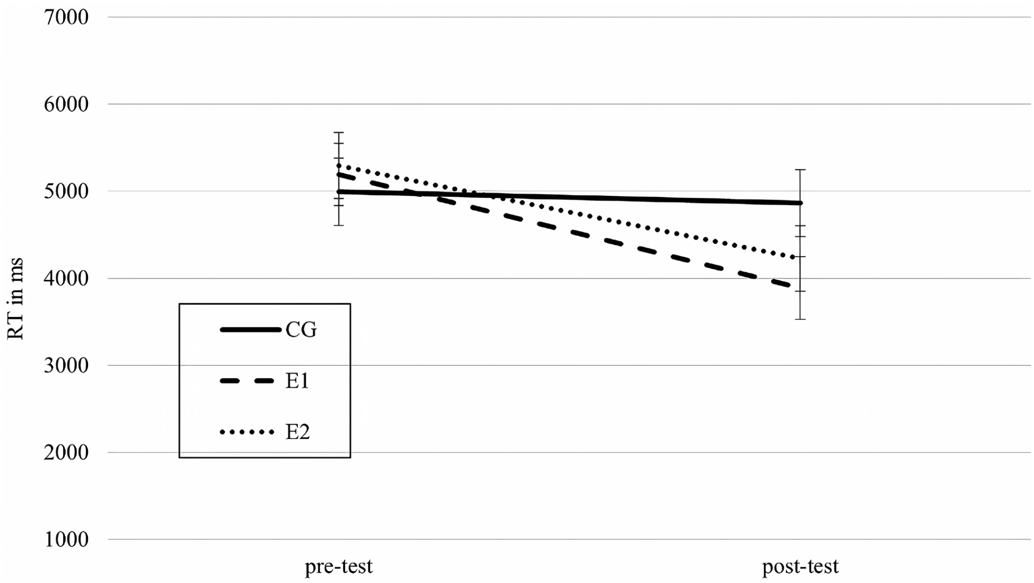

Figure 2. Performance in the reaction time experiment in the multiplication separated for pre- and post-test for all groups. Error bars indicate 1 standard error of the mean (SEM).

the interaction for the factors time and group was significant [$F(2, 61) = 7.33$, $p = 0.001$, $\eta_p^2 = 0.19$]. No significant interaction was found for group and operation [$F(2, 61) = 1.22$, $p = 0.301$, $\eta_p^2 = 0.04$]. Yet, the significant two-way interaction between time and operation [$F(1, 61) = 13.39$, $p = 0.001$, $\eta_p^2 = 0.18$] as well as the three-way interaction for the factors time, operation and group [$F(2, 61) = 3.25$, $p = 0.045$, $\eta_p^2 = 0.09$] indicate that solving multiplication problems improved significantly, compared to subtraction problems (cf. Figure 3). The slight improvement for subtraction problems can be explained by the fact that subtractions were still taught at school throughout the intervention with TIGRO-M. Parents and teachers were only instructed not to practice multiplication tables with the children before the intervention started, in order not to influence the effects of the training. There was no specific instruction for subtractions.

4.2. Stability of the Effects of Intervention

The analysis of stability for this intervention study is based on the quantity of the correct answer of the HRT 1-4 (Haffner et al., 2005) and obtains to the pre-, post- and follow-up test of the two experimental groups. The main effect of the factor time [$F(2, 78) = 40.26$, $p < 0.001$, $\eta_p^2 = 0.51$] shows a significant result, meaning a significant change of performance over time. The pairwise comparison for all test times shows that the pre- and post-, and the pre- and follow-up test differ significantly from another (pre- and post-test: $p < 0.001$; pre- and follow-up-test: $p < 0.001$). No significant difference in performance was found between post- and follow-up test ($p = 0.344$). These results clearly speak for the stability of the intervention (also Figure 4). The main effect factor group [$F(1, 39) = 0.25$, $p = 0.62$, $\eta_p^2 = 0.006$] was not significant, indicating no differences between the two intervention groups. Also the interaction between time and

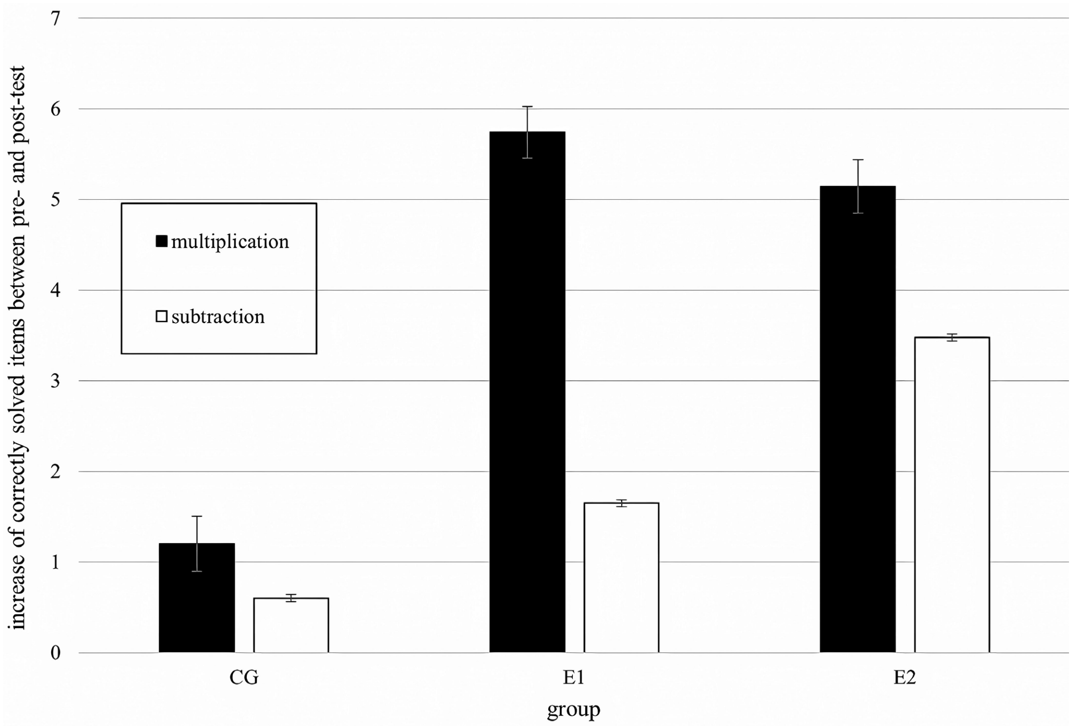

Figure 3. Specificity of the TIGRO-M program to multiplication fact improvement. Comparison between the subtest multiplication and subtest subtraction at the HRT after the intervention in all groups.

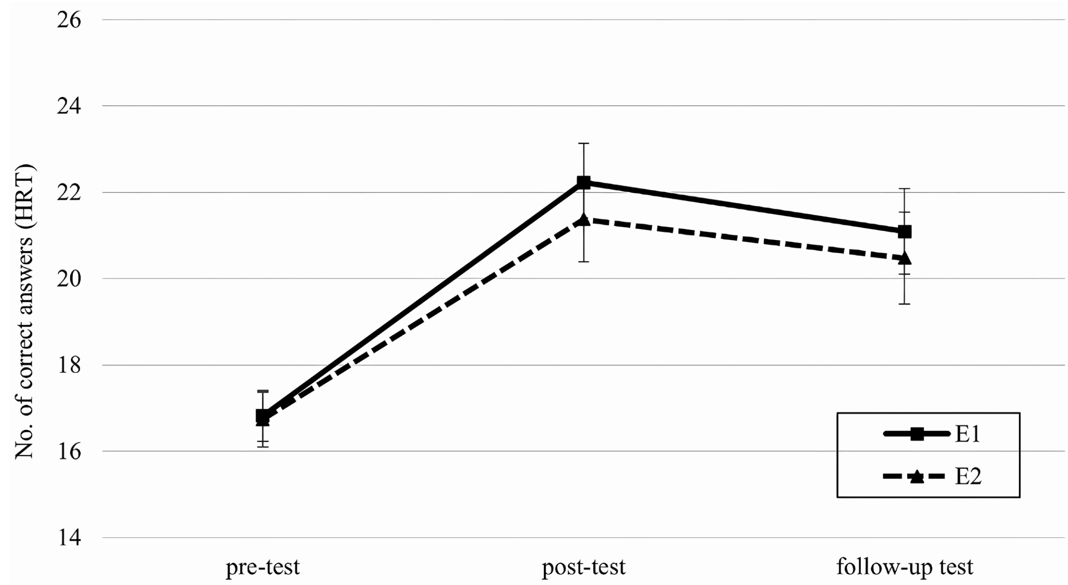

Figure 4. Stability of the attainment at the subtest multiplication at the HRT after the intervention of the two experimental groups.

group [$F(2, 78) = 0.23$, $p = 0.80$, $\eta_p^2 = 0.006$] provides a non-significant effect. The two experimental groups do not act differently during all three times of measurement.

The stability was analyzed with respect to response times on the computer test for all three test times (pre-, post- and follow-up test, see **Figure 5**). We found a significant main effect for the factor time [$F(2, 84) = 18.32$, $p < 0.001$, $\eta_p^2 = 0.30$]. Comparable to the results for correctly solved items, pairwise comparisons show that the pre- and follow-up, and the pre- and post-test differ significantly from one another (pre- and post-test: $p < 0.001$; pre- and follow-up-test: $p < 0.001$). The direct comparison between post- and follow-up test was not significant ($p = 0.878$). These results again show the stability of the intervention, also in matters of speed (for further information have a look at Figure 5). The main effect group [$F(1, 42) = 0.12$, $p = 0.74$, $\eta_p^2 = 0.003$] couldn't show a significant result. The two-way interaction for the factors time and group was not ($F(2, 84) = 0.231$, $p = 0.794$, $\eta_p^2 = 0.005$). The two experimental groups do not differ in their behavior during all three moments of measure.

4.3. Transfer-Effects

Since no significant differences between the two intervention groups regarding the effect of the intervention were found, we grouped them together for the following analyses regarding transfer effects. The 2 × 2 ANOVA, for response times on the computer tests, revealed the performance of transfer for the trained and not trained exercises after the training. We found a strong main effect for the factor time [$F(1, 43) = 22.09$, $p < 0.001$, $\eta_p^2 = 0.34$]. Children that received the intervention could clearly improve their performance in multiplication over time. The main effect of items (trained vs. untrained) [$F(1, 43) = 3.96$, $p = 0.053$, $\eta_p^2 = 0.08$] was only marginally significant, indicating that overall, trained facts were solved marginally faster than untrained multiplication problems. More importantly, the interaction for the factors time and items was not found to be significant [$F(1, 42) = 0.01$, $p = 0.97$, $\eta_p^2 = 0.00$], indicating that trained and untrained items show the same improvement over time (cf. Figure 6).

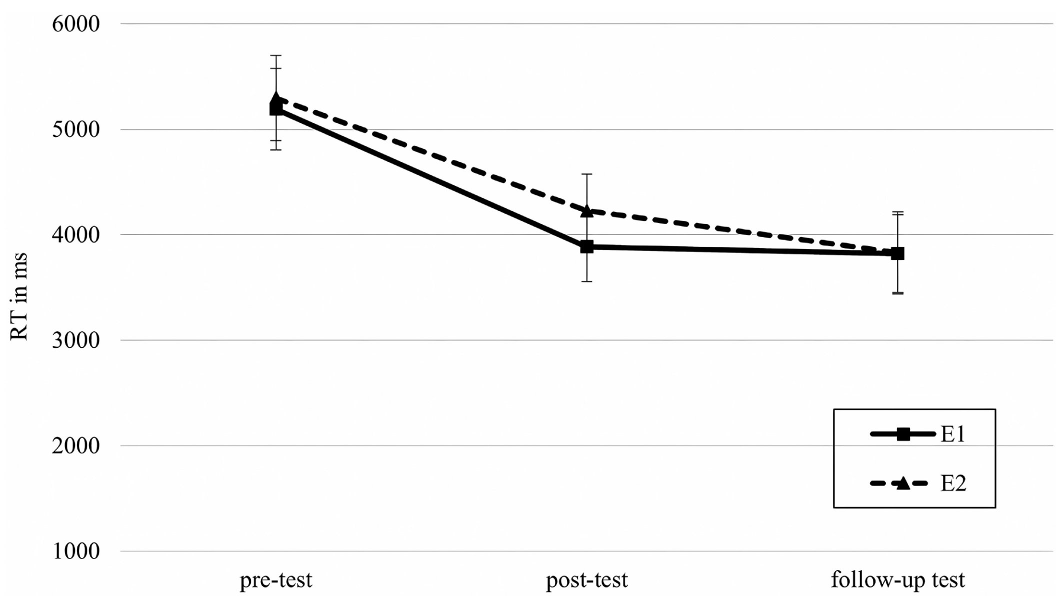

Figure 5. Stability of the attainment at the multiplication task during the response time experiment after the intervention for the experimental groups.

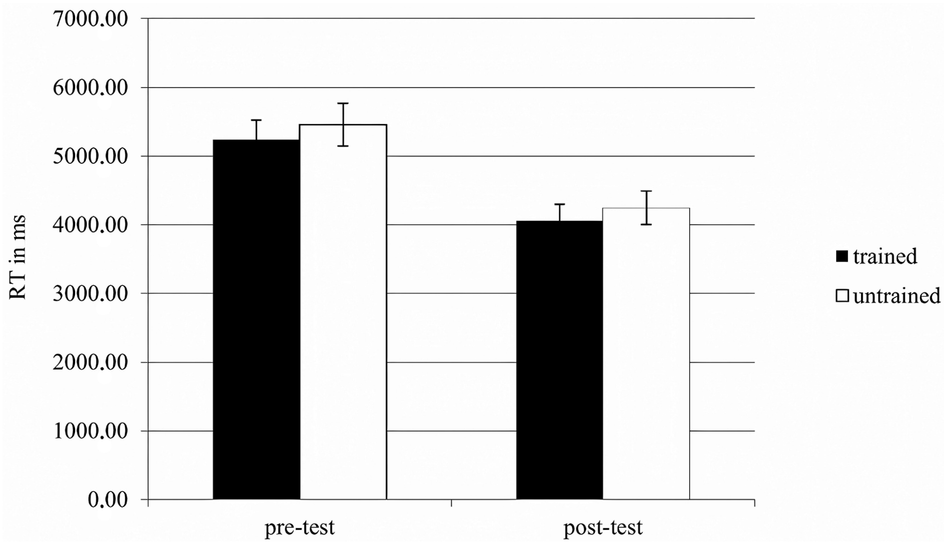

Figure 6. Transfer-effects of multiplication after the intervention between trained and untrained items

5. Discussion

The major goal of this study was to evaluate a new intervention program that trains multiplication facts for children with math difficulties. The distinctive characteristic of the program is that not only a pure memorization of the facts, but also a deep understanding of the process of multiplication is built. Additionally, we investigated whether number-color associations have a positive effect on the training process. We found that TIGRO-M has positive training effects, both in terms of correctly solved multiplication problems and in terms of response time when recognizing a correct result for a given multiplication problem. The positive effects of the intervention were specific to multiplications. However, not only did performance improve for trained problems, but clear transfer effects were observed for untrained multiplication problems. The stability of the achieved improvement after the intervention was also evident. Children showed a better quality of correct responses as well as response times even eight weeks after the end of the training phase, compared to before the training. Only an increase in the effectiveness of TIGRO-M by number-color associations could not be shown. All participating children, those who trained with selected colors and those who trained in black and white, showed the same improvements.

We firstly, and most importantly, aim to discuss the effectiveness of our intervention study TIGRO-M. Results demonstrated, that participating children from both experimental groups improved significantly in every domain, meaning accuracy and speed of solving multiplication problems, compared to the control group. Therefore, children were overall able to strengthen and improve their ability to solve multiplication problems. These positive intervention-effects can be particularly compared to the scientifically evaluated approach by Kroesbergen and van Luit (2003), who accentuated, that a direct and clear instruction is the most meaningful way to learn basic mathematical skills. Furthermore, the single-setting of this study fully catered to all requirements of each subject and meets with previous findings, that interventions show a higher effectiveness in a single-setting than in a group-setting (Kroesbergen & van Luit, 2003). Additionally, an adequate period of the intervention was proven to have a decisive role in training settings. At first glance, each of these factors may seem marginal, but in their entirety, they have a significant influence on the success of the training.

What sets TIGRO-M apart from others is the fact that it is based on three columns and takes a holistic view of the concept of multiplication. When considering the process of learning multiplication tables, the establishment of clearly structured and well-associated networks is the basis (Ashcraft, 1987). We especially considered this aspect with TIGRO-M, as we did not only focus on single memorization of multiplication facts. Instead, the intervention program is based on three columns, two of them being procedural and conceptual knowledge of multiplication. With TIGRO-M, the children i.a. use repeated addition to understand and internalize the concept of multiplication. The use of pictures of cubes or dominos for example involves an already known representation of quantity (2 × 3 = two times one cube with 3 dots on it). For building stable connections in long-term memory the children primarily have to work with the information and internalize the concept (Grolimund, 2012). The clear transfer effects show that these columns (procedural and conceptual knowledge) were very well adapted and internalized by the children. Trained facts were not just simply memorized, but a stable conceptual knowledge led to a stable associative network. Complementary operations with changed operands are stored in the same networks (cf. DeBrauwer & Fias, 2011). Previous findings (Grolimund, 2012; van Galen & Reitsma, 2011) support our assumptions, as children have the ability to recognize complementary operations and therefore practicing 6 × 3 leads to a better performance on the task 3 × 6 (see also Rickard & Bourne, 1996).

The one column, that is similar to other training programs is the automatization and storage of single facts. Repeating the contents of learning (TIGRO-M does that with different games) leads to learned materials migrating from working memory to long-term memory. Information that is thought about and repeated internally can build associations (Grolimund, 2012). Symonds and Chase (1992) postulated that automating is the most effective way for storing arithmetical knowledge.

Initially, we hypothesized that color-number associations would demonstrate additional benefits during automatization of multiplication facts. Yet, we did not find significant differences between the two experimental groups in terms of intervention effectiveness. Our findings, that the colorful representation of numbers does not further improve fact retrieval, are contrary to research by Domahs et al. (2004). They previously indicated that colors can be a key stimulus for storing arithmetic facts. However, Domahs and colleagues (2004) made their assumptions from a single-case-study. The tested individual was an older man with cerebral dysfunction, who obviously already learned arithmetic facts in his former life. Furthermore, the training set up differed to our intervention, as the training was carried out three times a week, as opposed to once a week in our study. Possibly, the higher frequency of training units in Domahs et al. (2004) led to a better connection of numbers and color.

Another explanation for the fact that no effect of the colors was shown, can be the circumstance of simulated synesthesia. With the use of color, we made an artificial link between the color and the contents. This link is not entirely new to our approach and is often used in other settings. It is not unusual that schools and special training approaches use colors in number contexts. The place value system is a perfect example for that, as colors are often used to tell the difference between different positions (e.g. the position for hundred is presented in red, decades in blue, and digits in green). In other cases, colors are used to present different calculation operations. This inconsistence use of colors might lead to a weakening of the expected effect. Unfortunately, the present study doesn't provide more information on this thought and therefore no further evidence can be shown in this direction. This limitation of the present study might be an interesting approach for further research.

In a further step, we were also able to prove that positive effects from TIGRO-M in terms of accuracy and speediness for solving multiplication problems, were stable over time. Children from both experimental groups (E1 and E2) could keep up their improved performance even eight weeks after the intervention phase, as no significant difference between post- and follow-up tests could be found. This speaks for an effective learning process with our intervention program. Ise et al. (2012) pointed out previously that most studies in this research field do not set value on follow-up measurement. However, demonstrating the gained knowledge right after the training must not be mixed up with the knowledge which is stored long-term. Our results show that participating children built stable and well-structured networks in their long-term memory and have not just memorized the multiplication facts without understanding. The conceptual design of the introduced training (TIGRO-M) is therefore perfectly suitable for an effective acquisition and long-term storing of multiplication and fact knowledge.

6. Conclusion and Practical Implications

In conclusion, the TIGRO-M's holistic approach to learning multiplication facts can be positively highlighted. In line with previous research, it can be concluded that the understanding of procedural and conceptual parts of multiplication can have a positive effect on building associative networks in long-term memory and, with the automatization of facts, can bring about a significant and long-term improvement in the performance of children with math difficulties. Therefore, we would like to emphasize again that all three components: procedural knowledge, conceptual knowledge and the automation of facts should be considered when promoting multiplication. Especially the conceptual understanding leads in our belief to a good connection between existing knowledge and thereby also achieves the stability over time that we were able to report. In addition, the reported transfer effects show that the knowledge was not only memorized, but can be transferred to new unlearned or related situations. This clearly speaks to the quality of this intervention program. Unfortunately, we could not confirm any additional positive effects of number-color association and thus could not empirically support our observations of beneficial effects on individual students.

As a prospect for the application of our intervention program in individual settings, it should be noted that the program can and should be adapted to individual needs. In the reported training, we have rigidly adhered to a given manual for the sake of comparability. In individual intervention, it should also be taken into account whether the children have problems in storing and remembering verbal facts or difficulties in understanding the multiplication itself (cf. Gomides et al., 2008). Therefore, the intervention should always be individually adapted, with a focus on the areas where support is most needed. We also expect the effectivity of the intervention to be even higher in a one-on-one setting with a trained professional for learning disorders/difficulties. Finally, the positive atmosphere of the training should be emphasized, which results from the structure of the intervention program (Pixner & Schlessing, 2021). The playful approach is particularly conducive to positive learning effects (cf. Sleeman et al., 2021).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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